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Exact Solution of the Nonlinear Problem of an Explosion in a Gas with Variable Initial Density 136354

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Let a finite energy E be liberated instantaneously at a point along a line or along a plane at the initial instant $t = 0$ in a gas at rest, i.e., an explosion has occurred. The energy E is computed per unit length in the case of a cylindrical charge and per unit area in the case of a plane charge [1]. A spherical, cylindrical or plane explosive shockwave is propagated through the gas. Unsteady one-dimensional gas motion with spherical, cylindrical or plane symmetry occurs outside the shockwave.

The initial pressure p_1 is constant, the initial gas density is variable and varies with distance from the center of the explosion as

$$\rho_1(r) = \frac{a(\gamma - 1)^2}{\gamma[\frac{1}{2}(\gamma + 1)]^{\beta-1}\left(\frac{r}{r^o}\right)^{\omega}\left[\left(\frac{r}{r^o}\right)^{\nu} + \frac{\nu(\gamma^2 - 1)}{2\sigma_\nu \gamma}\right]^{\beta}} \tag{1}$$

where γ is the ratio of the specific heats; a is a positive arbitrary constant; $\omega = \dfrac{\nu(3 - \gamma) + 2\gamma - 2}{\gamma + 1}$; $\beta = \dfrac{3\nu\gamma + 4 - \nu}{\nu(\gamma + 1)}$; $r^o = \left(\dfrac{E}{p_1}\right)^{1/\nu}$ is the dynamic length; $\nu = 3, 2, 1$ correspond to the spherical, cylindrical or plane wave cases; $\sigma_3 = 4\pi$; $\sigma_2 = 2\pi$; $\sigma_1 = 2$. It is seen from (1) that ρ_1 depends parametrically on the quantity γ and the dynamic length r^o .

One-dimensional adiabatic gas motions beyond the wave are described by the system of equations

$$\begin{gathered}
\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 \\
\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial v}{\partial r} + \frac{v(\nu - 1)}{r}\right) = 0 \\
\frac{\partial}{\partial t}\left(\frac{p}{\rho^\gamma}\right) + v\frac{\partial}{\partial r}\left(\frac{p}{\rho^\gamma}\right) = 0
\end{gathered} \tag{2}$$

where v is velocity; p is pressure; ρ is density. It is required to determine the dependence of the velocity, pressure and density of the gas on the linear coordinate r and the time t and also the dependence of the shockwave radius r_2 on the time.

The problem reduces to finding the solution of (2) with the abovementioned initial conditions and also with the boundary condition at the center of symmetry $v(0,t) = 0$ and the conditions on the front of the explosive wave which can be written thus [1]

$$(3) \qquad v_2 = \frac{2c}{\gamma + 1}(1 - q) \; ; \quad p_2 = \frac{p_1}{\gamma + 1}\,\frac{2\gamma - (\gamma - 1)q}{q} \; ; \quad \rho_2 = \frac{\rho_1(\gamma + 1)}{\gamma - 1 + 2q}$$

where $c = \frac{dr_2}{dt}$ is the shockwave velocity; $q = \frac{\gamma p_1}{\rho_1 c^2}$. Direct substitution convinces us that the solution of the problem formulated is given by

$$(4) \qquad v = \frac{r}{kt} \; ; \quad k = \frac{\nu(\gamma - 1) + 2}{2}$$

$$(5) \qquad p = \frac{p_1[kt]^{-\frac{\gamma\nu}{k}}}{\gamma + 1}\left\{\frac{4\gamma}{b(\gamma + 1)}[f(x)]^{-\frac{1}{2}(\gamma-1)} - (\gamma - 1)[f(x)]^{-\gamma}\right\}$$

$$(6) \qquad \rho = \frac{2p_1[kt]^{-\frac{(\nu-1)}{k}}}{r\nu(\gamma^2 - 1)}\,\frac{d}{dx}\left\{\frac{4\gamma}{b(\gamma + 1)}[f(x)]^{-\frac{1}{2}(\gamma-1)} - (\gamma - 1)[f(x)]^{-\gamma}\right\}$$

$$(7) \qquad \left(\frac{r_2}{r^o}\right)^{\nu} = \frac{\gamma^2 - 1}{2\gamma\sigma_\nu\left\{\frac{2}{b(\gamma + 1)}[kt]^{\frac{-\nu(\gamma+1)}{2k}} - 1\right\}}$$

where $x = r[kt]^{\frac{-1}{k}}$; $b = \left[\frac{\nu^2 p_1}{(\sigma_\nu r^o)^2 a}\right]^{\frac{1}{\beta-1}}$; $f(x) \geq 0$ is a function which never takes on negative values. The dependence of $f(x)$ is determined from

$$(8) \qquad \left(\frac{x}{r^o}\right)^{\nu} + \frac{\gamma^2 - 1}{2\sigma_\nu\gamma} f - \frac{2}{b(\gamma + 1)}\left(\frac{x}{r^o}\right)^{\nu} f^{\frac{1}{2}(\gamma-1)} = 0$$

The pressure variation directly outside the shockwave front is given by

$$p_2 = p_1\left[1 + \frac{\nu(\gamma - 1)}{\sigma_\nu}\left(\frac{r^o}{r^2}\right)^{\nu}\right]$$

The solution mentioned has been obtained from the exact solution of L. I. Sedov [2]. The method of constructing discontinuous solutions for this exact solution was developed by the author jointly with E. V. Riazanov.

We obtain the known solution [1] for the self-similar problem of a point explosion when the initial density obeys the law $\rho_1 = Ar^{-\omega}$, where A is a certain constant, from the solution we found in the particular case that $p_1 = 0$, $b = 0$.

The author is grateful to L. I. Sedov for formulating the problem and

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V. A. Steklov Math. Inst. June 18, 1957

References

1. L. I. SEDOV: Similarity and Dimensional Analysis in Mechanics, 4th Ed. 1957 (Translation exists)

2. L. I. SEDOV: DAN USSR, 90, No. 5 (1953) (Translation exists)